Kernel Machines and Reinforcement Learning, Workshop of the 23rd ICML, Pittsburgh

Equi-gradient TD Learning

TD(λ) on adaptative Basis Functions Network

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Introduction

- Kernel methods
- Reinforcement learning

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  new ***Regression method*** $\sim$ kernel method
  (regularized sample-based linear approximation)

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  Scheme for using it in ***TD(λ)***

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  equi-gradient TD(λ)

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- iterative gradient descent Osborne *et al.* 00

Equi-gradient descent

Least-Angle Regression Stagewise/laSSO

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- Generalization, simplification $\longrightarrow$ ***Equi-gradient descent***

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$\widehat{\mathbf{y}} = \mathbf{\Phi}_a \mathbf{w}_a$

$$\text{minimize } (\mathbf{y} - \mathbf{\Phi}_a \mathbf{w}_a)^2 + \lambda \sum_i |w_i|$$

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Regularization path

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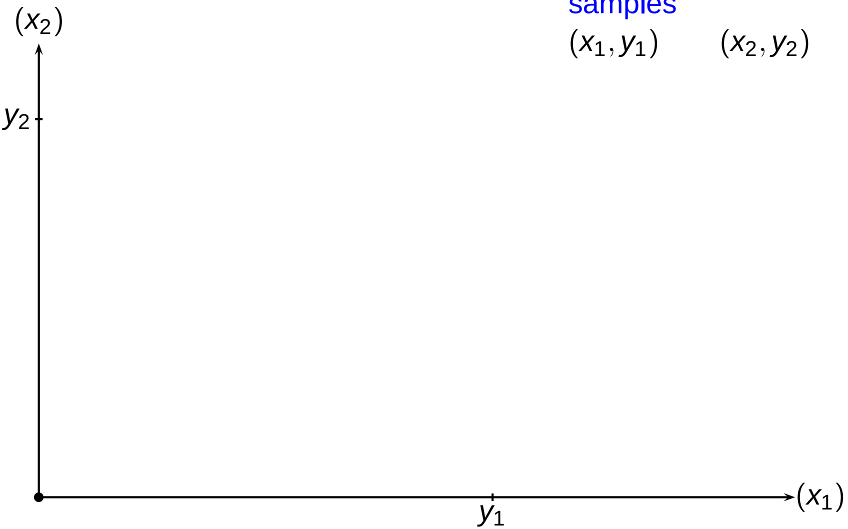
samples

(x_1, y_1) (x_2, y_2)

Regularization path



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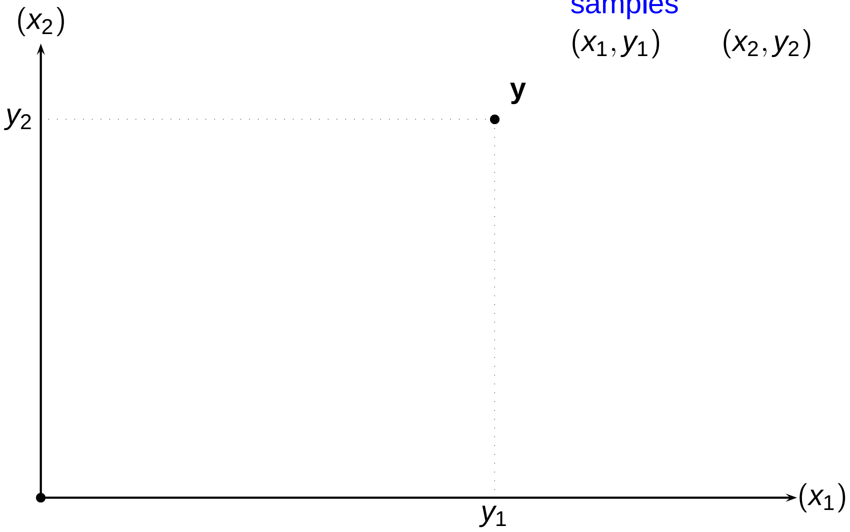

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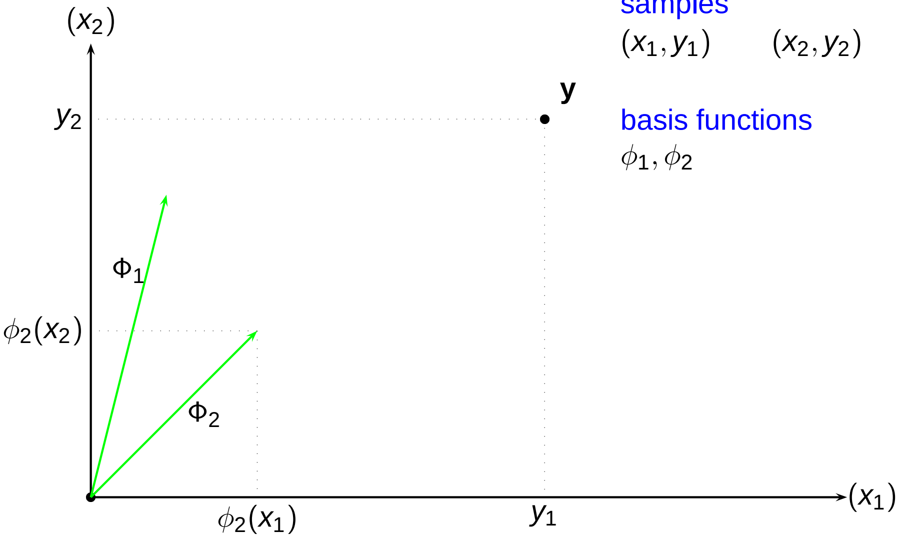

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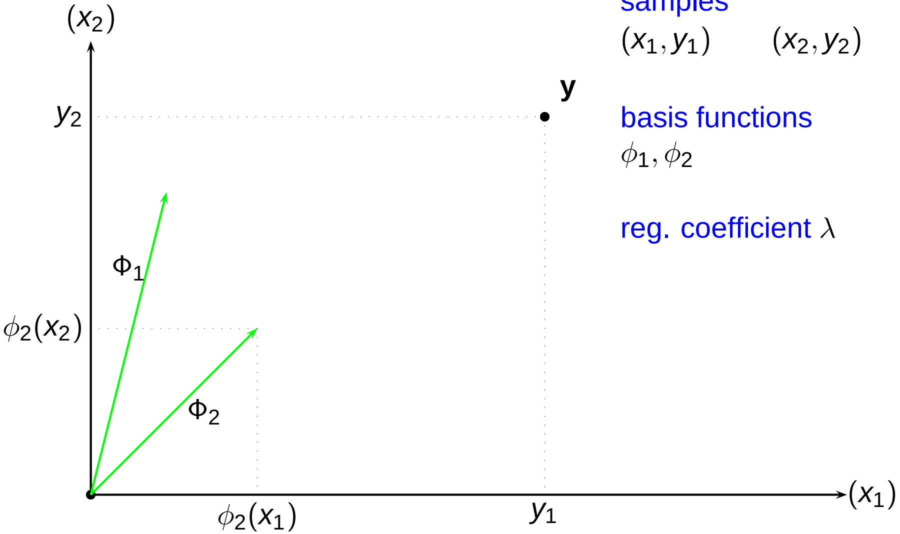

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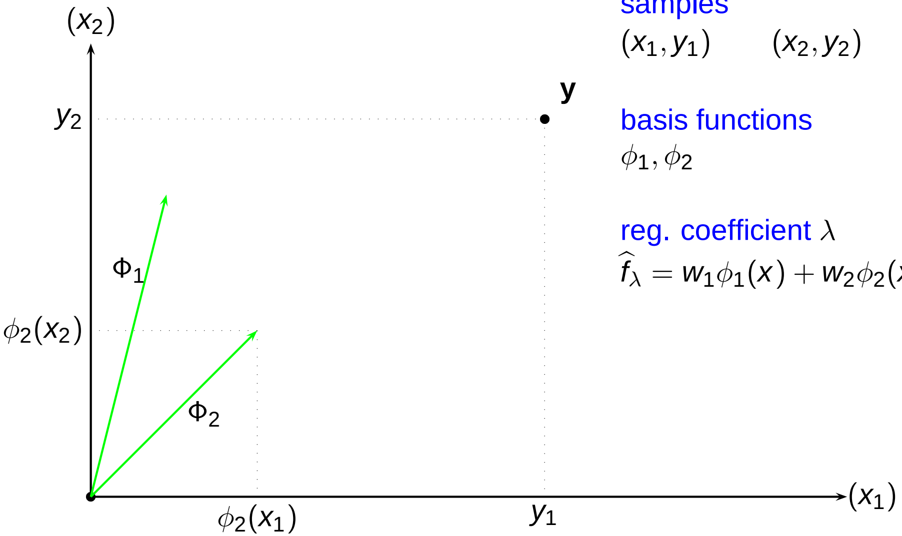

samples

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$\widehat{f}_\lambda = w_1\phi_1(x) + w_2\phi_2(x)$

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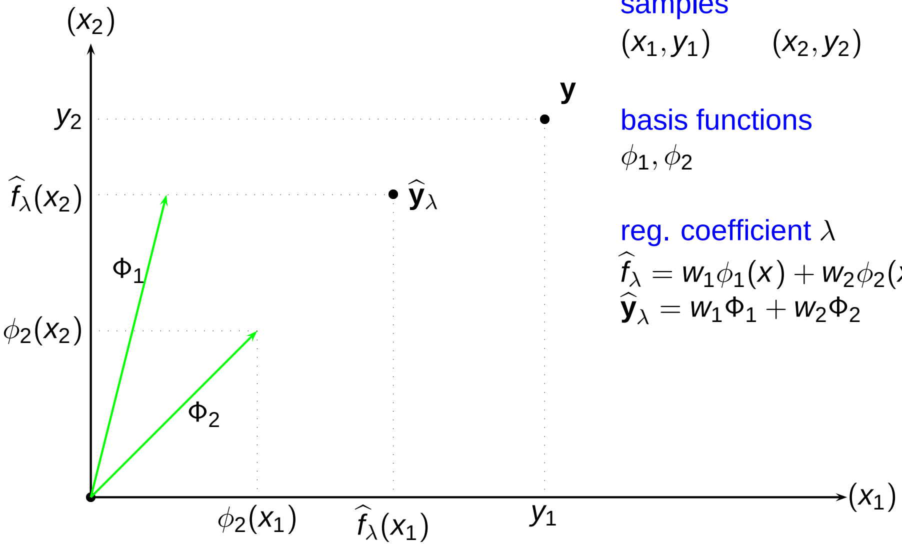

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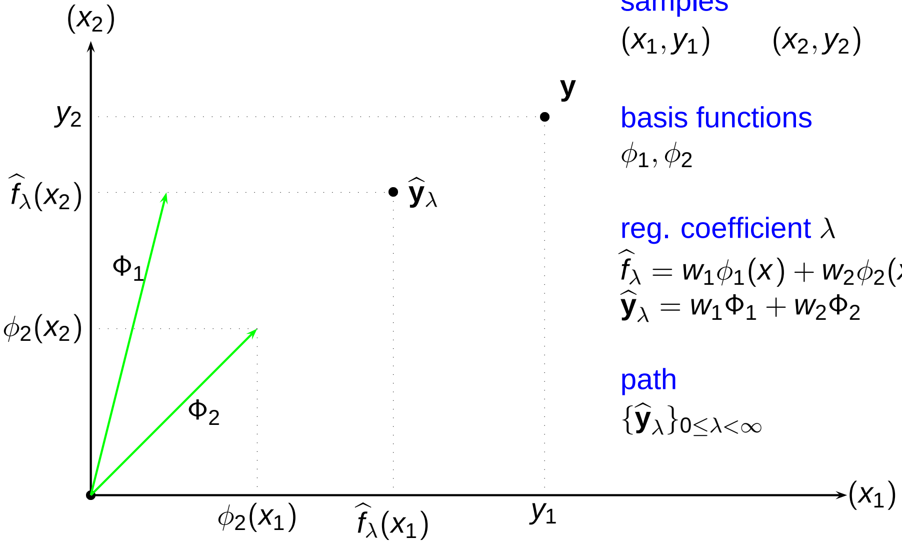

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path

$\{\widehat{\mathbf{y}}_\lambda\}_{0\leq\lambda<\infty}$

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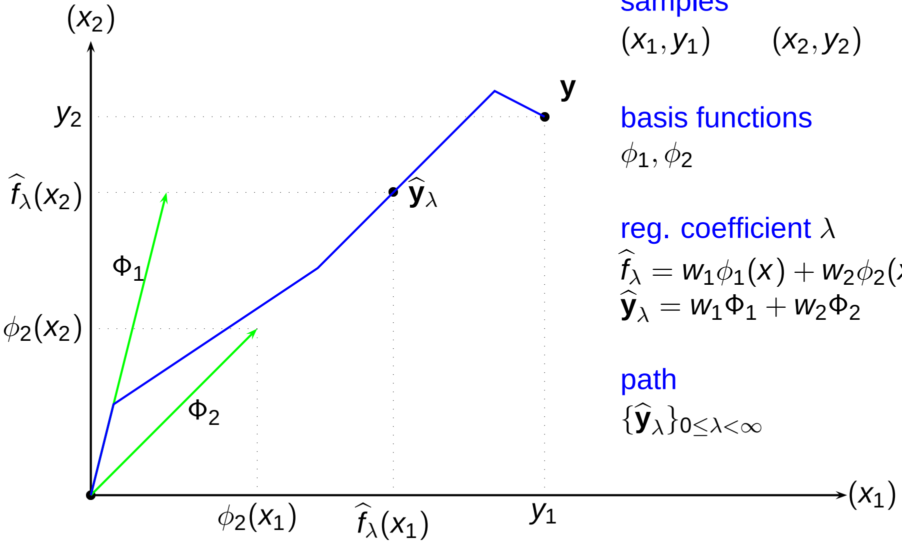

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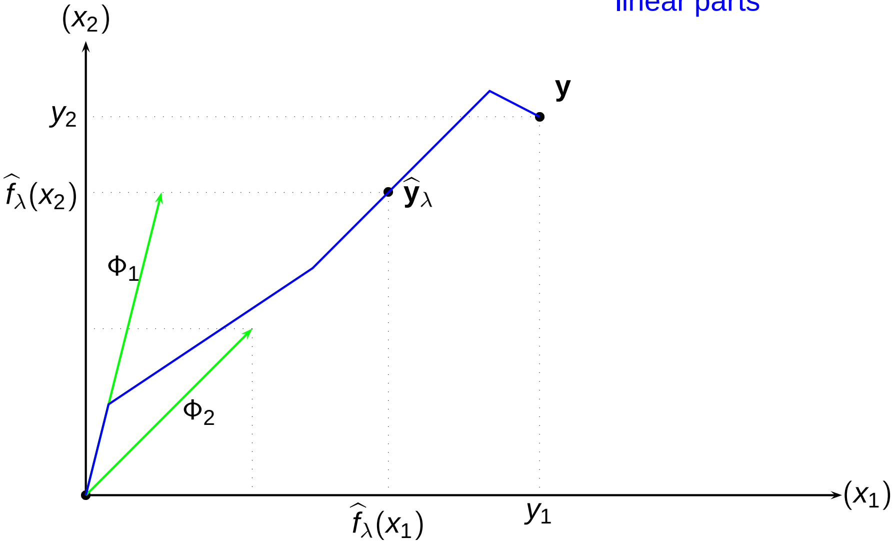
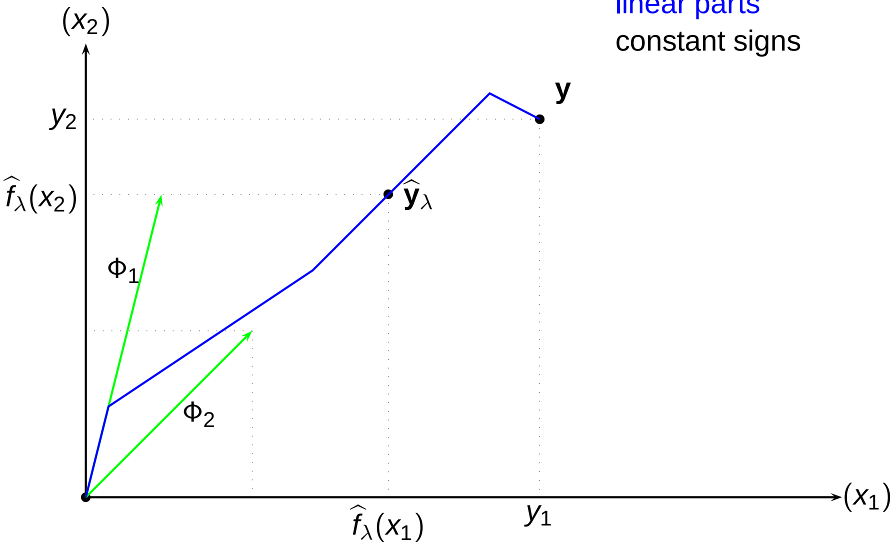
linear parts



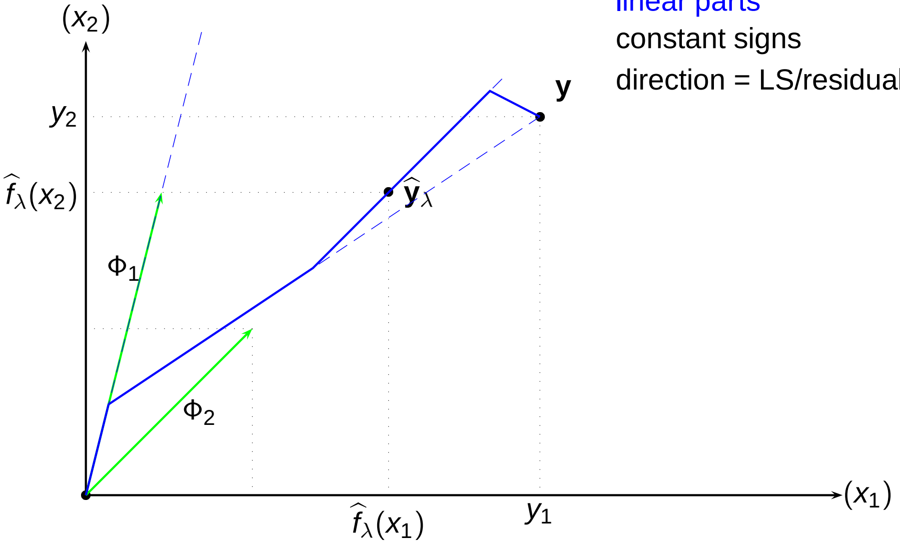
Regularization path



linear parts

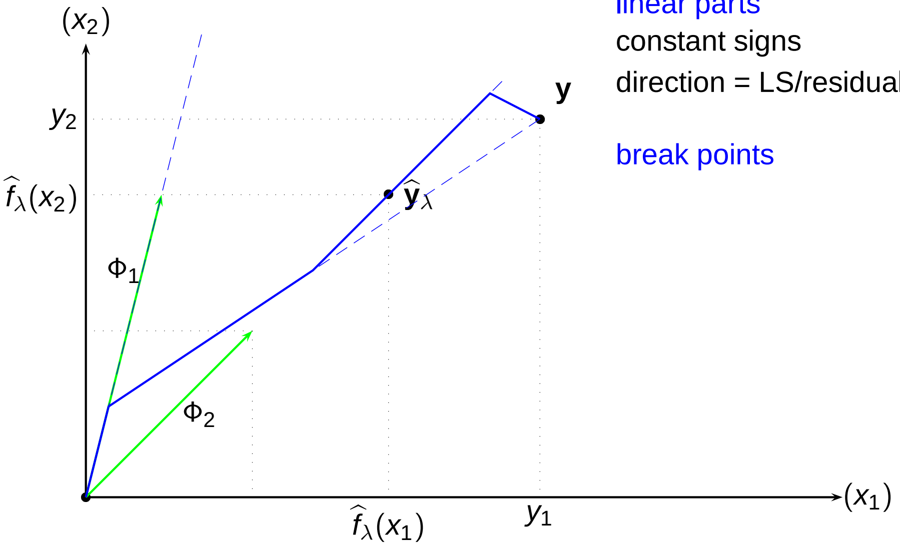
constant signs

Regularization path



linear parts
constant signs
direction = LS/residual

Regularization path



linear parts
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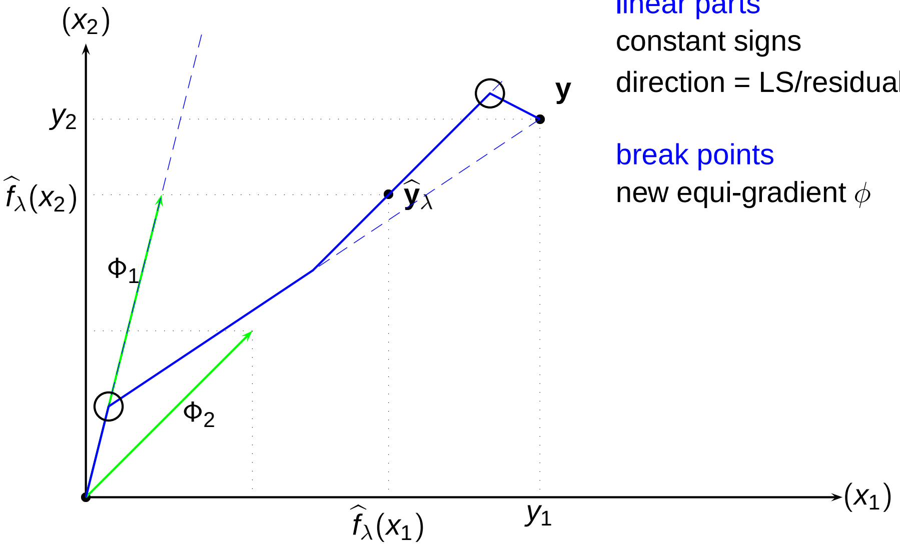
direction = LS/residual

break points

Regularization path



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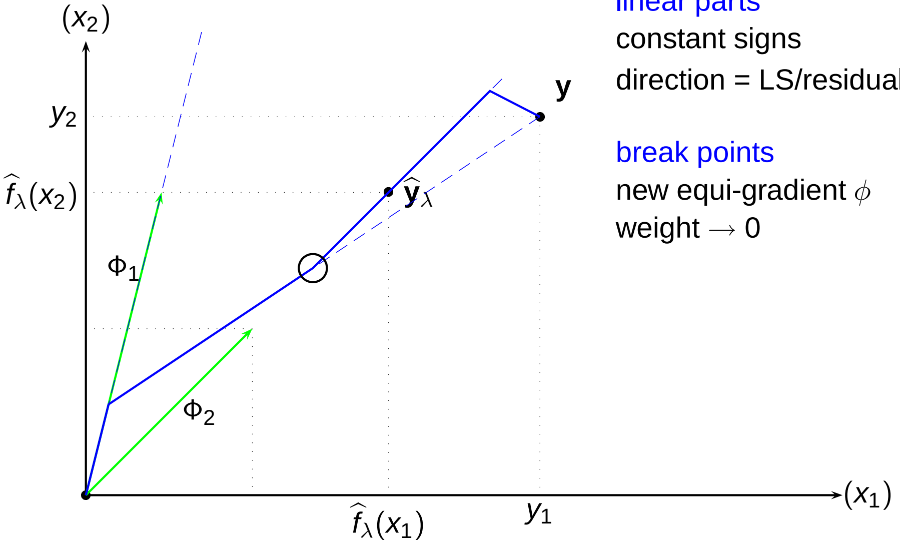
new equi-gradient ϕ

Regularization path



linear parts
constant signs
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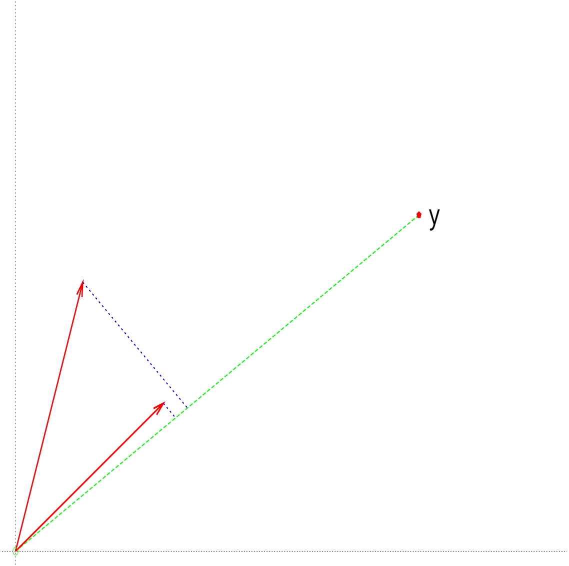
new equi-gradient ϕ
weight $\rightarrow$ 0

Regularization path



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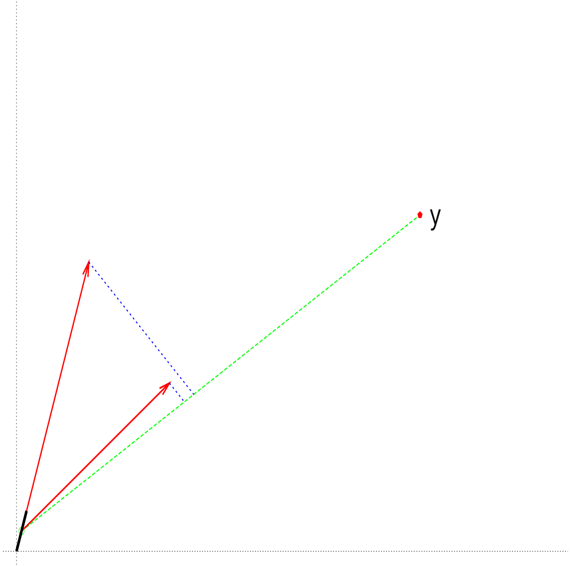

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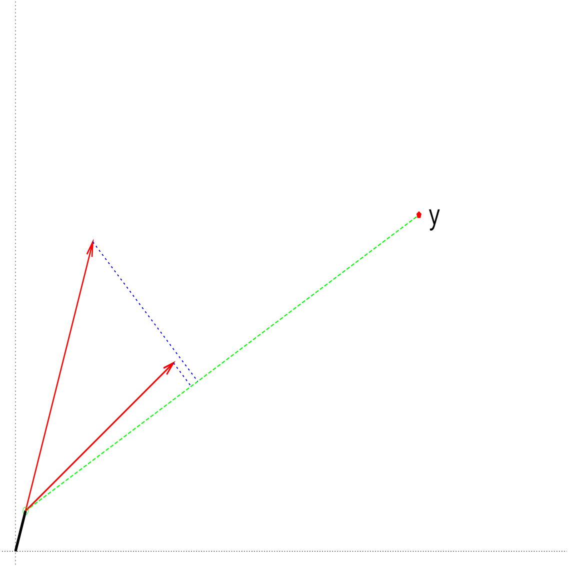

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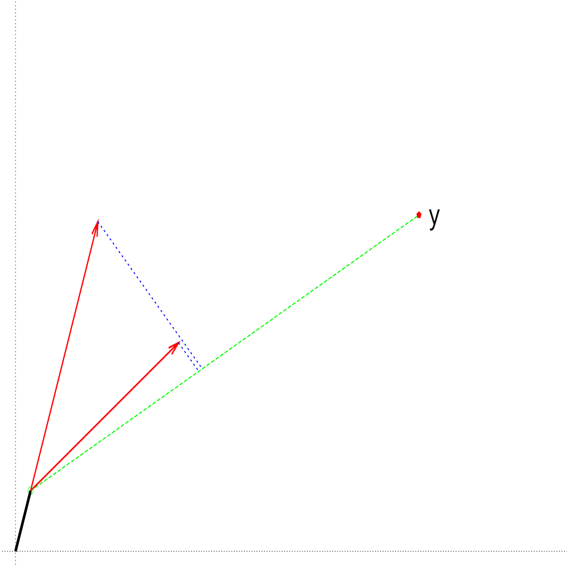

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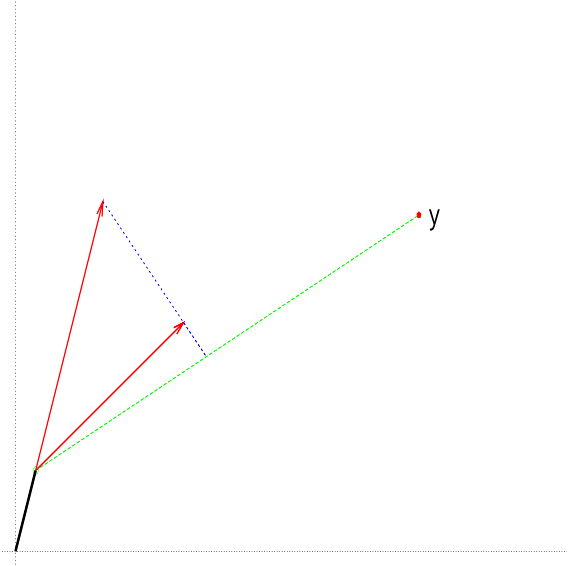

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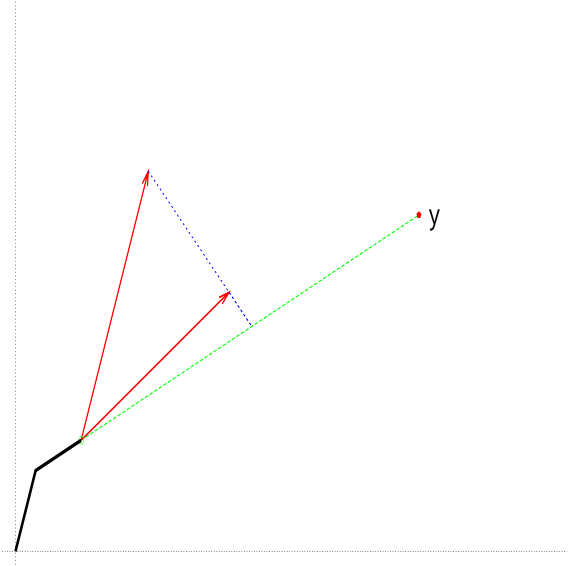

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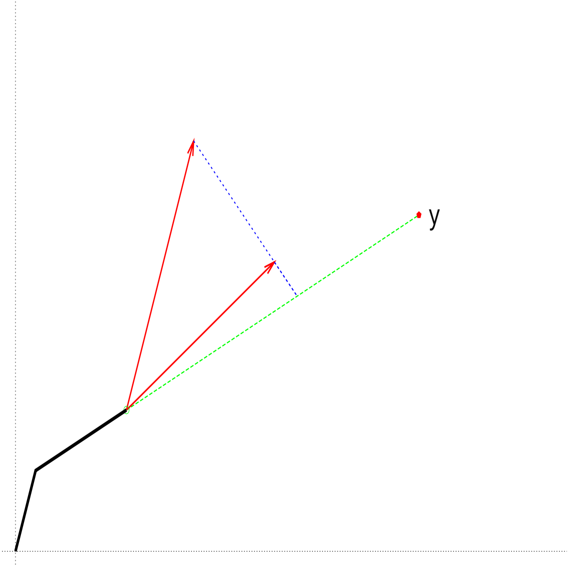

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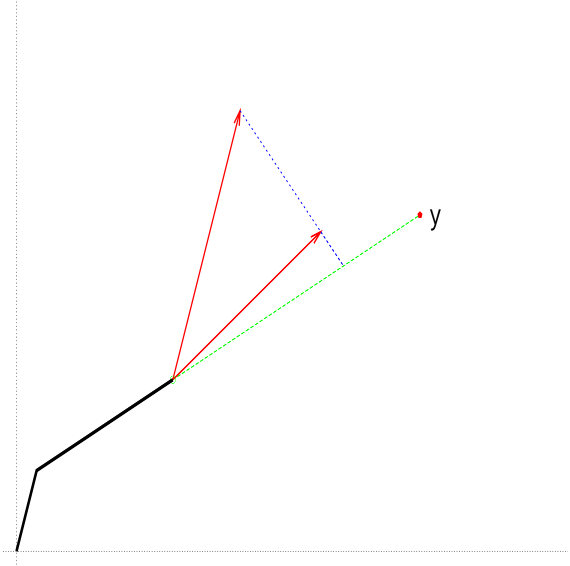

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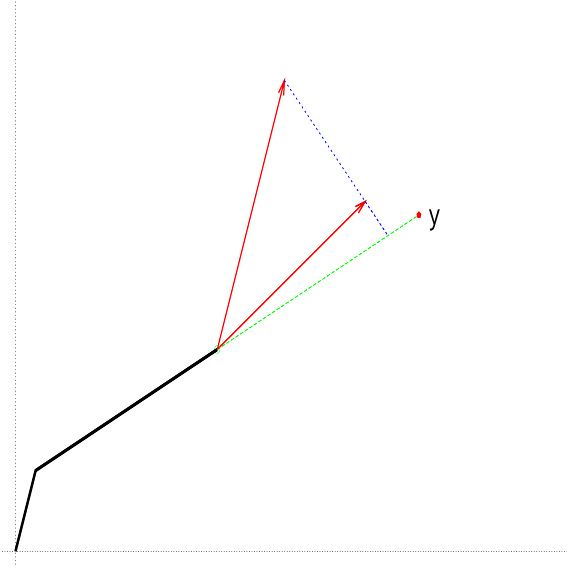

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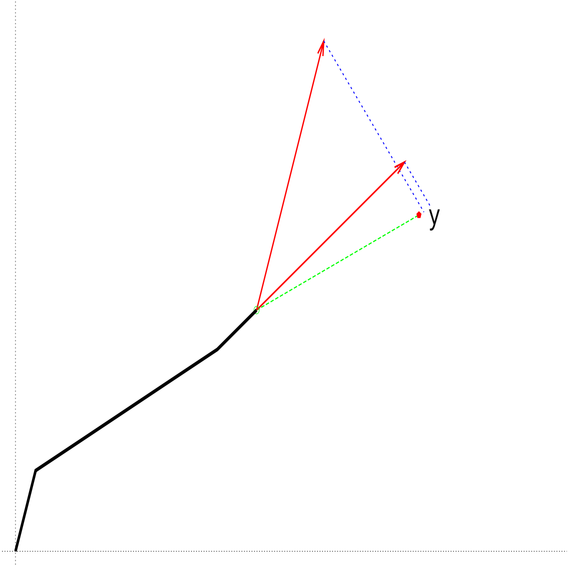

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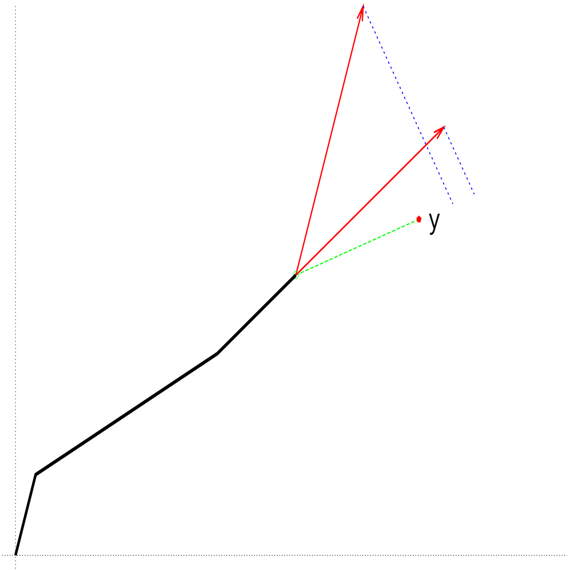

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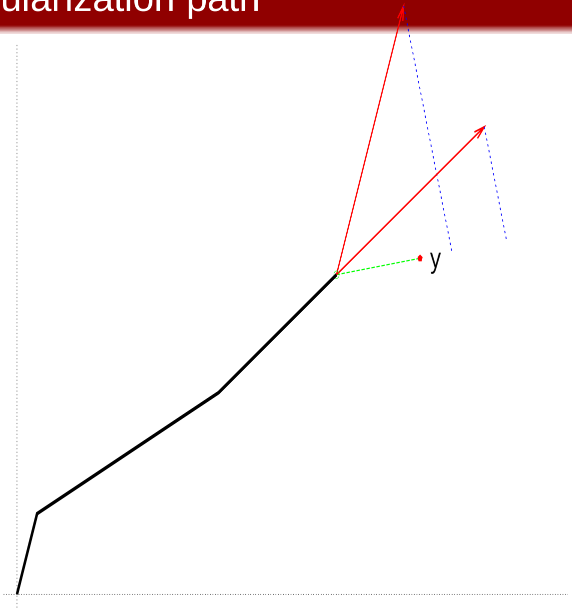

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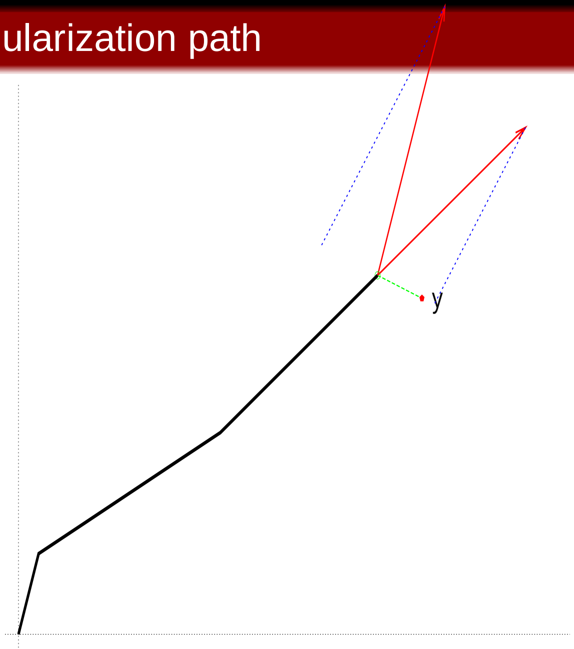
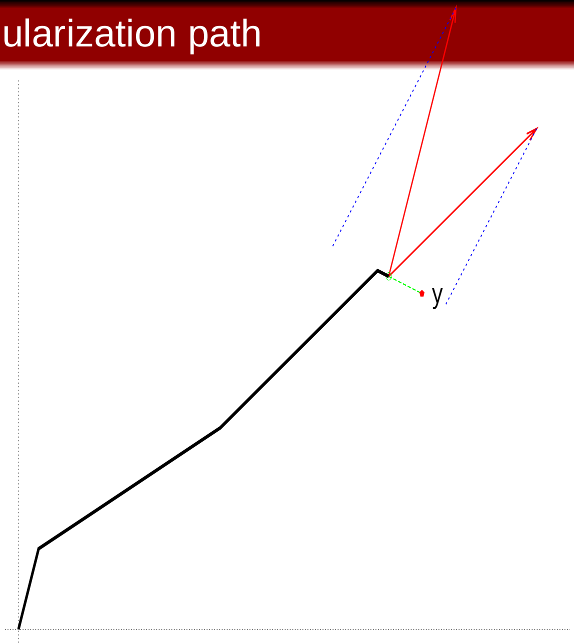
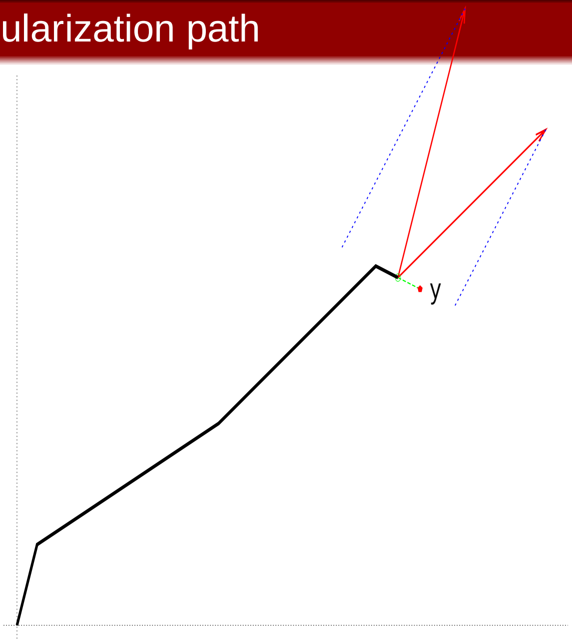


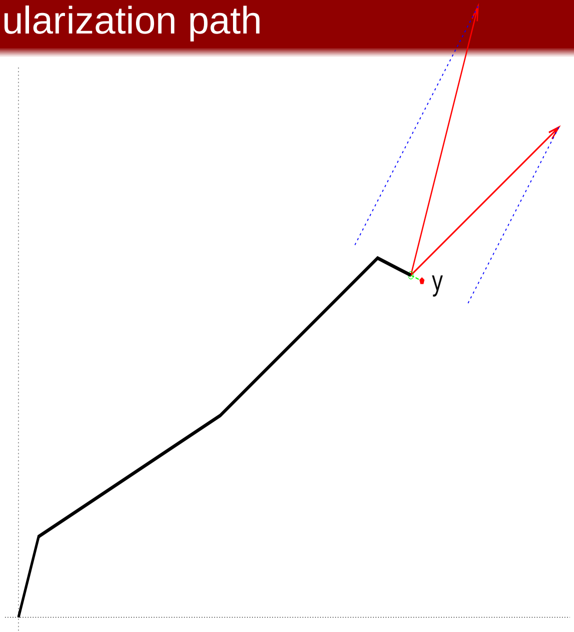
Regularization path



Regularization path



Regularization path



Regularization path

Complexity

Each step (activation/deactivation) is $O(|\mathcal{D}| + |\mathcal{A}|^2)$

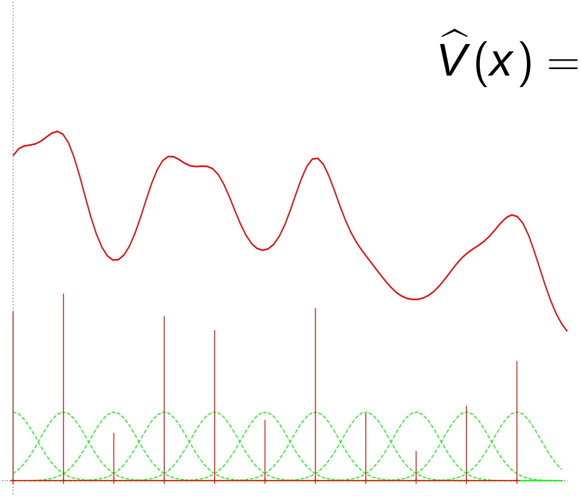
- linear in dictionary size
- quadratic in number of active basis functions

Demo

Reinforcement Learning

- TD(λ)
- continuous state space
- discrete time
- updates after each trajectory

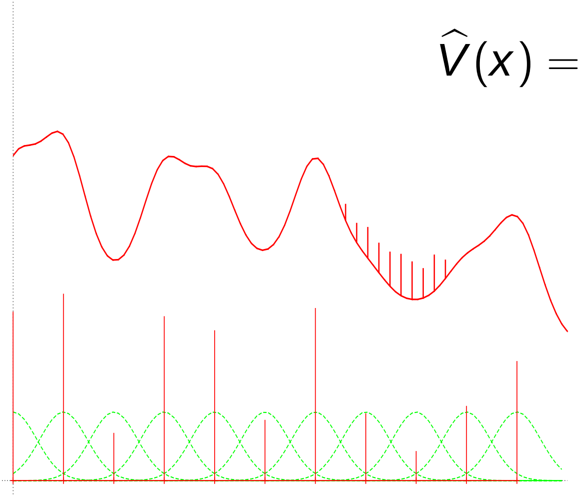
Radial Basis Functions Network



$$\widehat{V}(x) = \sum w_i \phi_i(x)$$

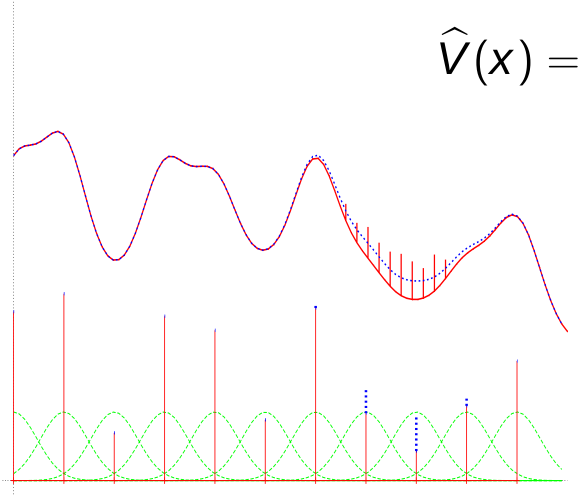
Radial Basis Functions Network

$$\widehat{V}(x) = \sum w_i \phi_i(x)$$

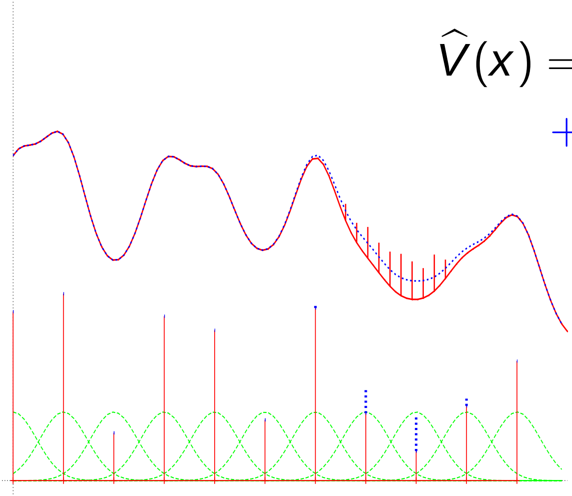


Update

$$\widehat{V}(x) = \sum (w_i + \Delta_{w_i}) \phi_i(x)$$

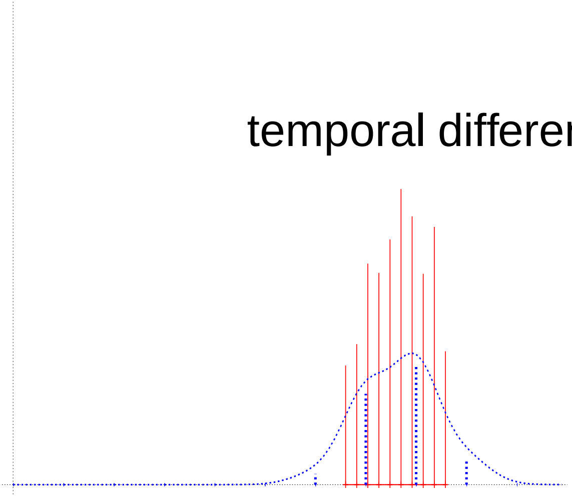


Update



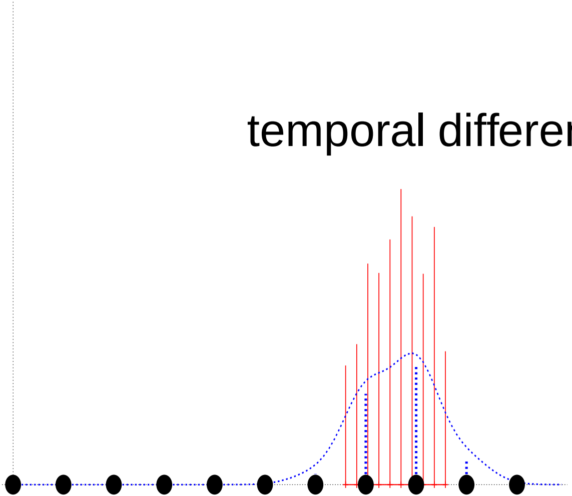
$$\hat{V}(x) = \sum w_i \phi_i(x) + \sum \Delta_{w_i} \phi_i(x)$$

Independant regression

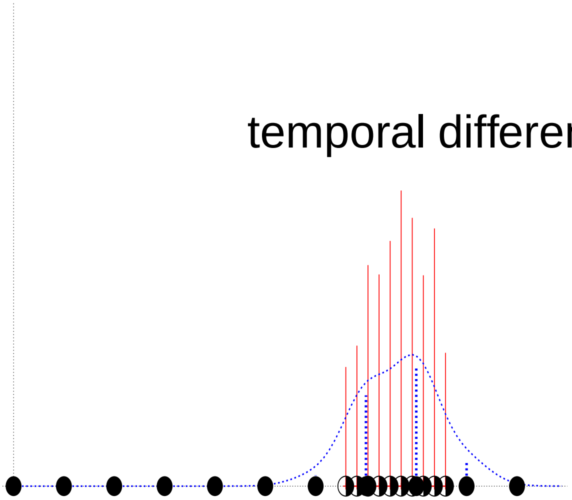


Gradient descent on fixed basis network

temporal differences



Equi-gradient descent on extended network



Temporal regularization

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Continuously add basis functions?!

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- Put a preference on existing basis functions

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- Put a preference on existing basis functions
- Remove zero-weighted basis functions

Temporal regularization

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- Put a preference on existing basis functions
  $\phi_i \leftarrow \rho\phi_i$
- Remove zero-weighted basis functions

Temporal regularization

Continuously add basis functions?!

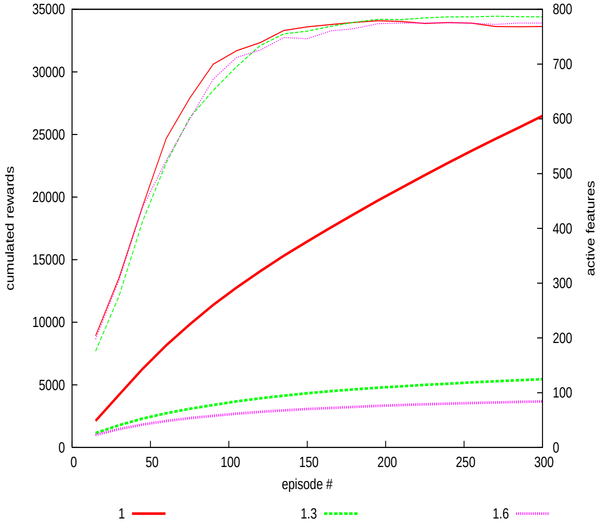
- Put a preference on existing basis functions
  $\phi_i \leftarrow \rho\phi_i \Rightarrow w_i \leftarrow \frac{1}{\rho}w_i$
- Remove zero-weighted basis functions

Preliminary simple experiments

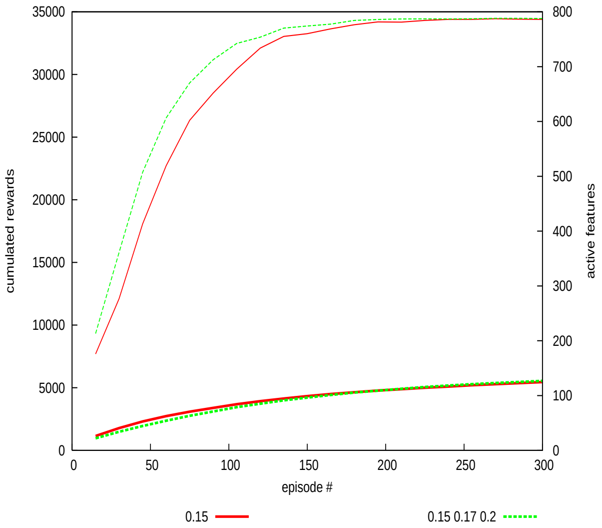
Preliminary simple experiments

- Inverted pendulum
- Gaussian basis functions on normalized state space
- Updates after each episode
- Stopping EG descents at $|\widehat{\mathbf{y}}|^2 = 70\%|\mathbf{y}|^2$

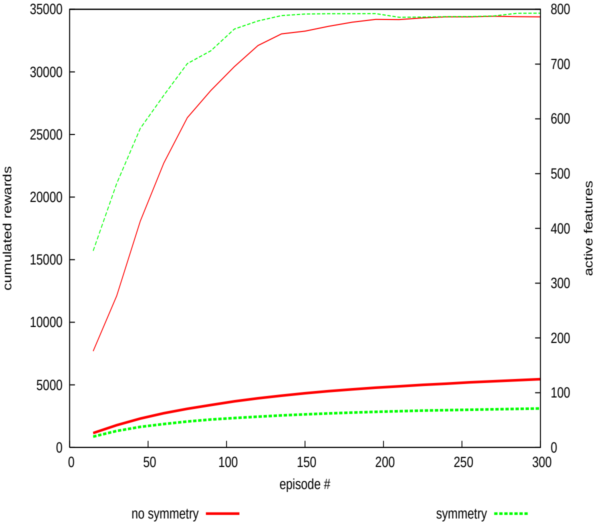
Experiments: preference



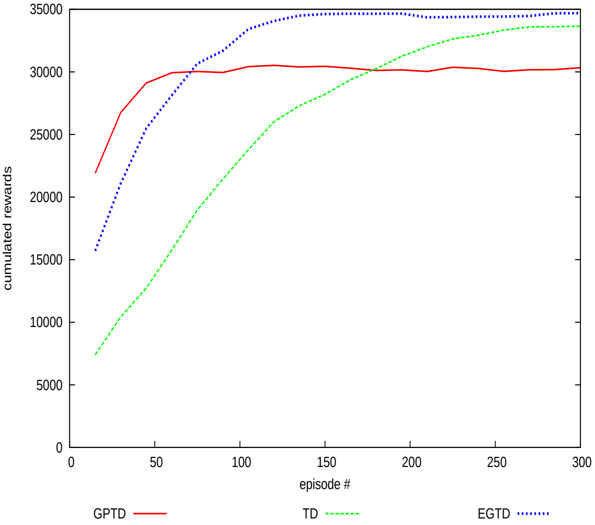
Experiments: multi-kernels



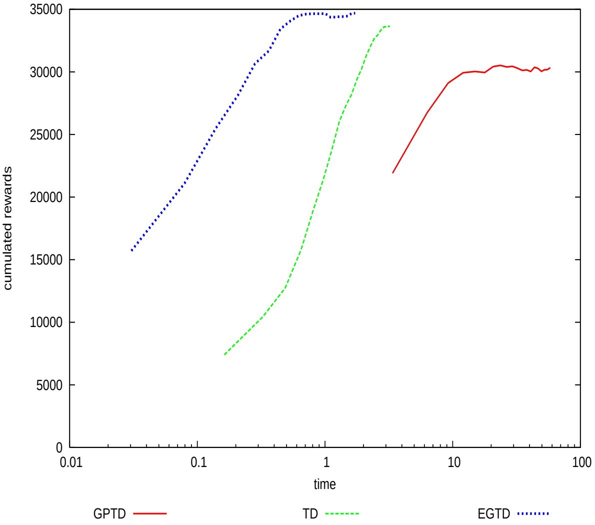
Experiments: symmetry



Experiments: comparison



Experiments: comparison



Conclusion

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Summary:

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- ***efficient*** and ***easy*** way to select basis functions in TD(λ)

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Perspectives:

Conclusion

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- ***robust***, no unintuitive parameters

Perspectives:

- experiments on other problems

Conclusion

Summary:

- ***efficient*** and ***easy*** way to select basis functions in TD(λ)
- ***robust***, no unintuitive parameters

Perspectives:

- experiments on other problems
- automatically build basis function dictionary based on topology, TD variance, . . . (wavelets, low-dimensional projections,. . .)

Take home message

- Feature selection is easy!
- Basic use of it in RL $\longrightarrow$ efficient & easy-to-tune TD(λ)