

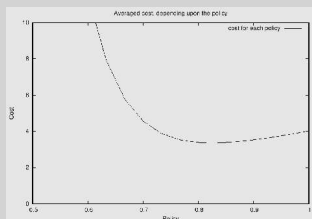
Taylor-based pseudo-metrics for random process fitting in dynamic programming : expected loss minimization and risk management.

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Stochastic optimization is the research of x optimizing $E C(x,A)$, the expectation of $C(x,A)$, where A is a random variable. Typically $C(x,a)$ is the cost related to a strategy x which faces the realization a of the random process.

Example in dimension one of the expected cost for a given distribution A , in function of the different strategies. Being too short is much worse than being too long.



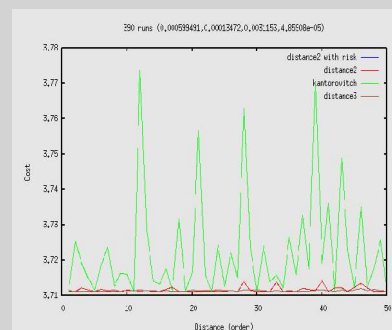
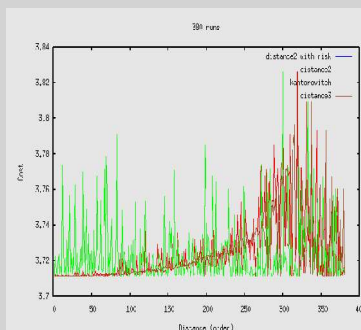
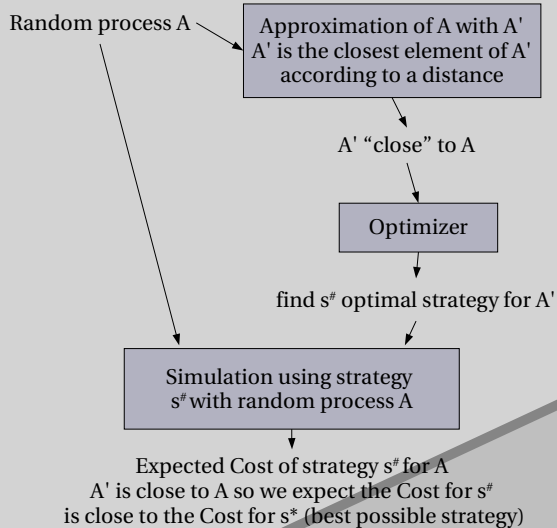
So it is crucial to have good estimations of distance between two random process. A classical method is the use of the Kantorovitch-Rubinstein distance. We propose other (pseudo-)distances, based upon refined inequalities : We want the alea A' which minimise $\sup_s |E(C(s,A)) - E(C(s,A'))|$. Using a Taylor expansion we propose these « distances »

$$distance_1(\pi(A), s_0) = |E_A \nabla_A C(s_0, A)(\pi(A) - A)|$$

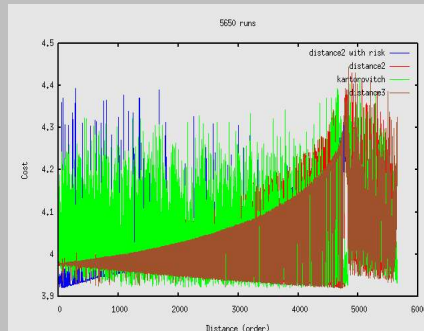
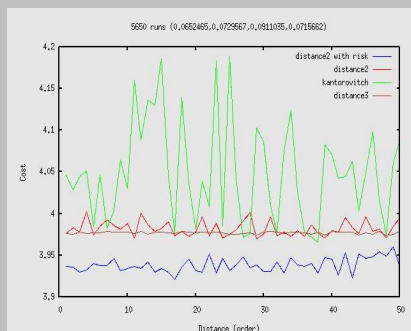
$$distance_1'(\pi(A), s_0) = E_A |\nabla_A C(s_0, A)(\pi(A) - A)|$$

$$distance_2(\pi(A), s_0) = \|E_A \nabla_A C(s_0, A)(\pi(A) - A) + \frac{1}{2}(\pi(A) - A)' H_A C(s_0, A)(\pi(A) - A)\|$$

Typical paradigm for solving this kind of problems:



Results in dimension 1 without noise. X-coordinate is the rank of A' according to the considered distance. For a "good" distance we expect that small distances are associated to a small cost. The second graphic is a zoom on the smallest distances between A and $\pi(A)=A'$. We can see that the Kantorovitch-Rubinstein distance gives very bad results because the cost is not correlated to the distance.



Same graphics than above but with noisy datasets and with adapted our adapted mesures.

