Taylor-based pseudo-metrics for random process fitting in dynamic programming : expected loss minimization and risk management.

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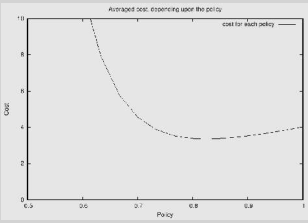
Stochastic optimization is the research of x optimizing $E\,C(x,A)$, the expectation of $C(x,A)$, where A is a random variable. Typically $C(x,a)$ is the cost related to a strategy x which faces the realization a of the random process.

Example in dimension one of the expected cost for a given distribution A, in function of the differents strategies. Being too short is much worse than being too long.



Typical paradigm for solving this kind of problems:

Random process A → Approximation of A with A'
A' is the closest element of A' according to a distance

A' “close” to A

Optimizer

find s# optimal strategy for A'

Simulation using strategy s# with random process A

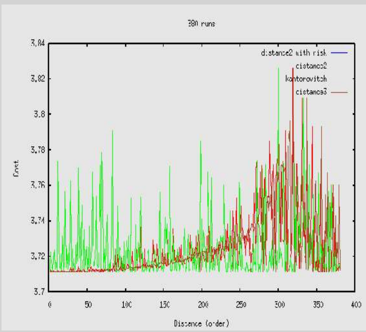
Expected Cost of strategy s# for A
A' is close to A so we expect the Cost for s#
is close to the Cost for s* (best possible strategy)

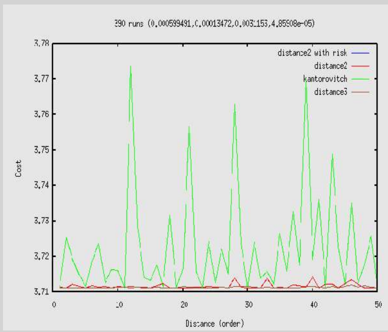
So it is crucial to have good estamitions of distance beetwen two random process. A classical method is the use of the Kantorovitch-Rubinstein distance. We propose other (pseudo-)distances, based upon refined inequalities :
We want the alea A' which minimise $\sup_s |E(C(s,A))-E(C(s,A'))|$.
Using a taylor expansion we propopose theses«dis tances»

$$distance_1(\pi(A),s_0)=|E_A \nabla_A C(s_{0,}A)(\pi(A)-A)|$$

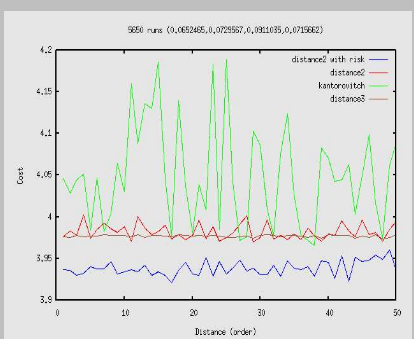
$$distance_1'(\pi(A),s_0)=E_A|\nabla_A C(s_{0,}A)(\pi(A)-A)|$$

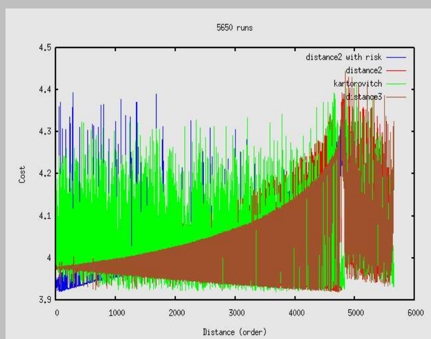
$$distance_2(\pi(A),s_0)=||E_A \nabla_A C(s_{0,}A)(\pi(A)-A) + \frac{1}{2}(\pi(A)-A)^t H_A C(s_{0,}A)(\pi(A)-A)||$$





Results in dimension 1 without noise. X-coordinate is the rank of A' according to the considered distance. For a ``good'' distance we expect that small distances are associated to a small cost. The second graphic is a zoom on the smallest distances between A and $\pi(A)=A'$. We can see that the Kantorovitch-Rubinstein distance gives very bad results because the cost is not correlated to the distance.





Same graphics than above but with noisy datasets and with adapted our adapted mesures.

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