Introduction to Game Theory

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Outline

Prisoner's dilemma

	Prisoner B Stays Silent	**Prisoner B Betrays**
Prisoner A Stays Silent	Each serves 6 months	Prisoner A: 10 years Prisoner B: goes free
Prisoner A Betrays	Prisoner A: goes free Prisoner B: 10 years	Each serves 5 years

Nash Equilibrium

A set of strategies is a Nash equilibrium if no player can do better by unilaterally changing his or her strategy.

Does this equilibrium always exists ? Let's try to be a prisonner !

Nash

Theorem

Nash proved that if we allow mixed strategies, then every n-player game in which every player can choose from finitely many strategies admits at least one Nash equilibrium.

- How to compute the nash's strategies ?
  Usually iterating a virtual game, with best responses to opponent's strategie.
- Are these equilibrium stables ? Depends if it exists some equivalent strategies for one player.

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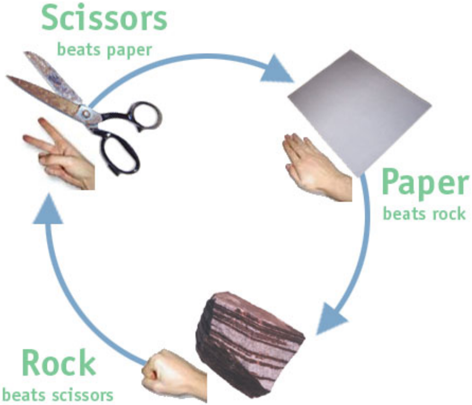
Rock-Paper-Scissors



In video games

Combat or strategy-based video games often feature rock-paper-scissors-like cycles in their characters' or units' effectiveness against others.

⇒Leads to interesting games, which requires to quickly adapt. Can be improved with some signals before attacks.

Lizards (Uta stansburiana)

- Orange males are strongest and do not form strong pair bonds; instead, they fight blue-throated males for their females.
- Blue males are middle-sized and form strong pair bonds. While they are outcompeted by orange-throated males, they can defend against yellow-throated ones.
- Yellow males are smallest, but looks like females. So they can approach orange-throated males (though not the blue-throated specimens) and mate while the orange-throats fights.

Artificial RoShamBo player

- What is the Nash Equilibrium ?
  ⇒Random Play.
- How play against a Nash player ?
  ⇒You should play Nash too. But here this is unimportant.
- Nash is optimal in a certain sense, but can we do better than it if we face a non Nash Player ?
  ⇒Hopefully yes, because playing against Nash players is boring for humans.

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Dive Into The Code

1. Code Human (You) vs Nash.
2. Try Nash against “good old Rock” (always rock).
3. Compute a best response against a constant strategy. What is the strategy obtained by randomly choose one of the best response strategies at each turn ?
4. Work on a way to defeat a strategy playing too much of one category
5. Defeats a cyclic strategy (or a strategy which always want to beat last opponent play).
6. Think on a statistical way to guess the opponent play.

Further Improvements

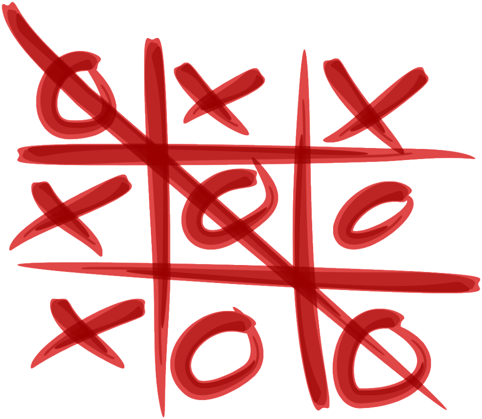
- Implements a Meta-Strategy that choice among a large set of strategies (this is easy because we have the results for all the strategies whatever we play).
- Do some history matching in addition to frequency matching.
- Introduce the Nash strategy in order to limit losses against stronger opponent (which mandatory exists ☹).

⇒ Have a look to Iocaine

http://dan.egnor.name/iocaine.html

But do not forget, you only take advantage from bad players and in this kind of video games best players are "turtles" ☺

Tic-Tac-Toe



Minimax

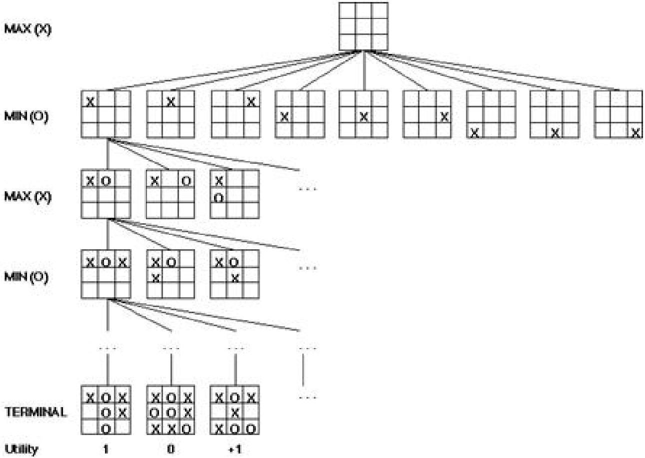
Theorem

For every two-person, zero-sum game with finite strategies, there exists a value V and a mixed strategy for each player, such that

- *Given player 2's strategy, the best payoff possible for player 1 is V,*
- *Given player 1's strategy, the best payoff possible for player 2 is -V.*

means that for most of games there exists some optimal strategies and some predefined results

Tic-Tac-Toe Tree



Bigger trees

1. What would be the time necessary to compute the optimal strategy of a game like chess ?
   something like 10^{120} possible games (around 40 moves a game) remember the number of atoms in the universe (10^{80})
2. How to go faster ?
   heuristics and pruning…

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How to code all that's stuff ?

Recursivity

Simplified poker

Each player must pay 1$ to the pot to receive a value uniformly drawn in $[0, 1]$. They keep this value secret (incomplete information game). At this point, you have 2 choices :

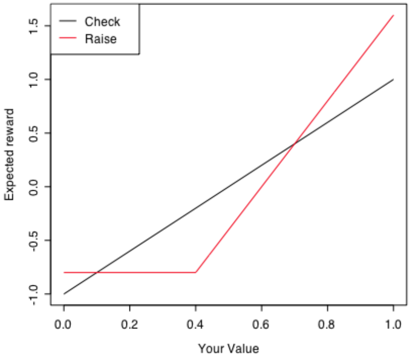
- Check - Don't add money to the pot
- Raise - Add 1$ to the pot

And now your opponent can choice :

- Fold - he loses but don't add any money to the pot
- Check - equalises the bets and reveal the secret numbers. The higest win the pot.

Opponent modelling and bluff

Now consider you know you opponent will Check if it's free or if his secret number is greater than 0.4. We can compute the expected value of our two actions as a function of our secret number :



Challenges

- Go
- Poker
- Multiplayer games.